DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF COPENHAGEN

CENTRE FOR STOCHASTIC GEOMETRY AND ADVANCED BIOIMAGING

Modelling Time Evolution of Medical Images

Line Kühnel, Stefan Sommer & Alexis Arnaudon
kuhnel@di.ku.dk, sommer@di.ku.dk, alexis.arnaudon@imperial.ac.uk

https://slides.com/linekuhnel/deck/live#/

Motivation

Our goal is to model the evolution and variation of medical images. As a motivational example consider the problem of modelling the evolution of childrens brains (Fig. 1). In general, brains develop similar features such as ventricles and the corpus callosum. However, no human being are alike so each subject will evolve in a different manner giving a variation in the evolution of the brains. A big focus has been on modelling the population trend. We present a model estimating not only the population trend, but also the variation of the data during evolution. This will be pursuid by considering stochastic image registration.

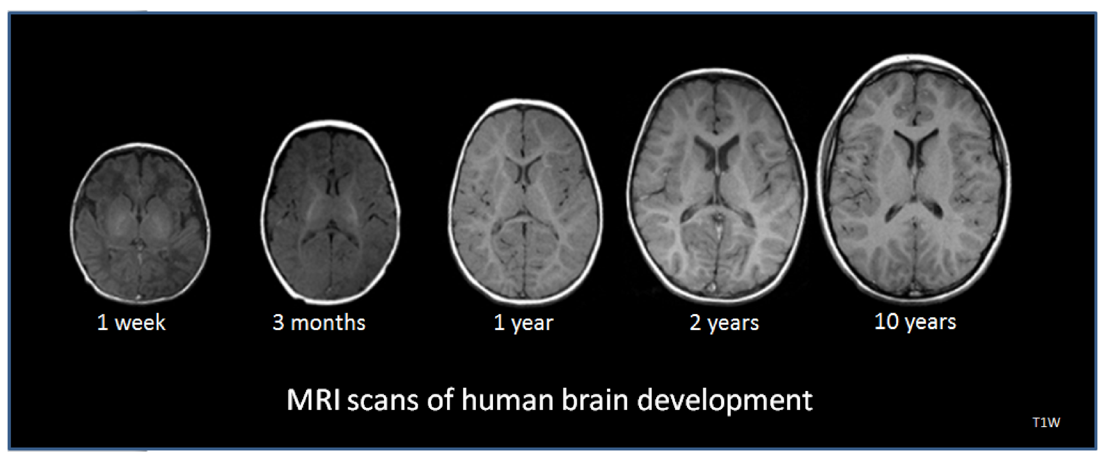

Figure 1: Example of the brain evolution of a child from 1 week to 10 years.

Euclidean Equivalence

Let us describe the presented model based on Euclidean data. Consider growth curves of children shown in Fig. 3. In this example, we seek to model the time evolution of the distribution of heights of children. Let x_{ij} be the j'th time observation of subject i, β be the population trend, α_i the subject individual intercept and ϵ_{ij} the noise, then

$$x_{ij} = \alpha_i + \beta t_{ij} + \varepsilon_{ij}, \; t_{ij} \in (0, 20]$$

The goal is to estimate the noise ε_{ij} and population trend β.

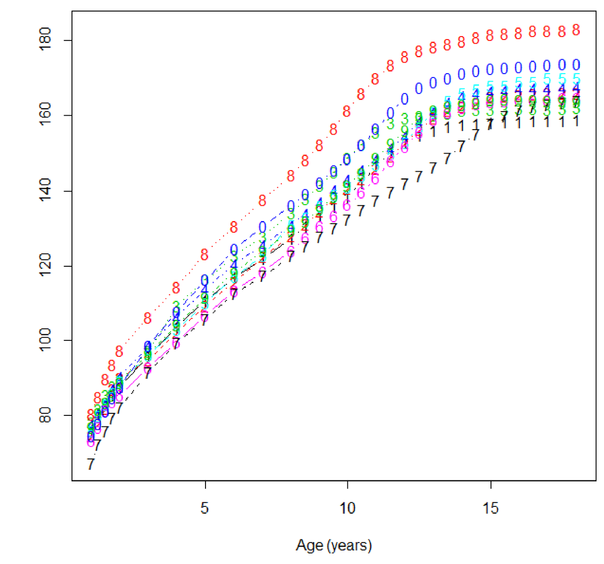

Figure 3: Growth curves of children

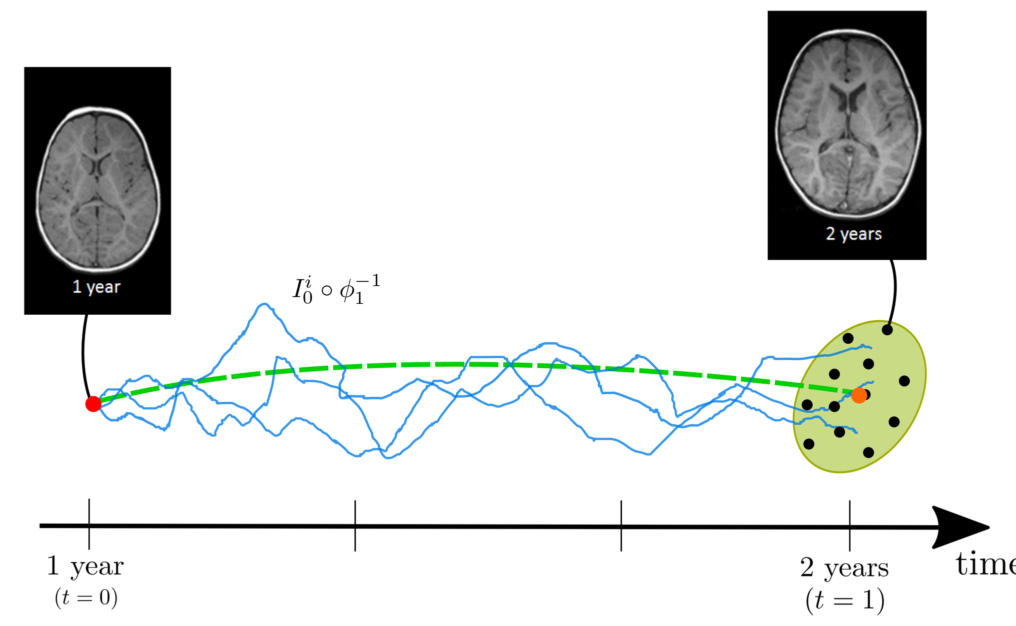

Figure 2: Modelling the evolution of variation of medical images by stochastic processes.

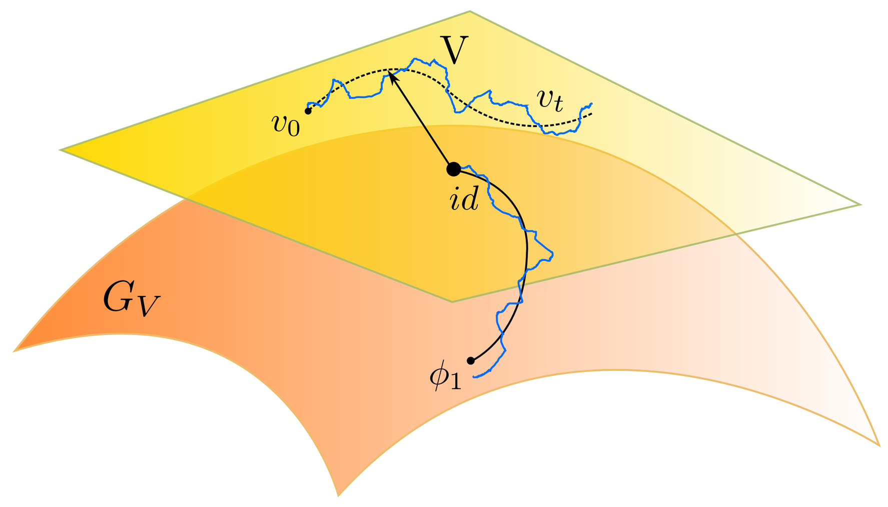

Figure 4: Visualization of the dependency between ϕ^{-1} and v. G_V is the space of diffeomorphisms solving (2).

Stochastic Shooting

Consider the situation where n subjects have been observed at time $t = 0$ and $t = 1$. The presented model can given an initial image I_0^i define a stochastic deformation of I_0^i which transition distribution describes the uncertainty of the subject at $t = 1$. The stochastic deformations of images are defined in the LDDMM framework where deformations minimize,

$$E(v_0) = \frac{1}{2}\|v_0\|_V^2 + \sum_{i=1}^{n} \|I_0^i \circ \phi_1^{-1} - I_1^i\|_{L^2}^2, \qquad (1)$$

for a diffeomorphism ϕ_1^{-1} and an initial point v_0 for a time-varying velocity field v_t. In the deterministic LDDMM model the diffeomorphisms minimizing (1) is the endpoint of a flow solving (2) with v_t solving (3) (the deterministic part). We will consider the stochastic deformation given by a similar construction as the deterministic version,

$$d\phi_t^{-1} = -D\phi_t^{-1} v_t dt - \sum_{k=1}^{d} D\phi_t^{-1} \sigma_k \circ_S dB_t^k \qquad (2)$$

where the time-varying velocity field solves,

$$dv_t = -ad_{v_t}^{\dagger} v_t dt - \sum_{k=1}^{d} ad_{\sigma_k}^{\dagger} v_t \circ_S dB_t^k \qquad (3)$$

Here B_t^k denotes Brownian motions and σ_k are noise fields on the domain of the images. For a visualization of the model see Fig. 2. When parameters of the noise fields σ_k are known, the model can sample observations for new subjects at time $t = 1$ and hence summarize the uncertainty and variation of the evolution of the subject.

Method of Moments

The Fokker-Planck equation (left (4)) describes the evolution of the density function $p(x, t)$ for a Itô stochastic process X_t. Based on the Fokker-Planck equation, the evolution of moments of the distribution of X_t is given by,

$$\frac{\partial}{\partial t} p(x, t) = \mathcal{L}^* p(x, t), \qquad \frac{d}{dt}\langle h(X_t)\rangle = \langle \mathcal{L} h(X_t)\rangle, \qquad (4)$$

where $\mathcal{L}$ is the Kolmogorov-operator, which can be found based on Itô's lemma. The goal is to match the moments of the data with the transition moments of the stochastic processes, ϕ_t^{-1}, v_t. That is: first solving the moment ODE in (4) results in the transition moment of ϕ^{-1} or v, e.g. the first moment $\langle \phi_1^{-1} \rangle$, which is matched to the first moment of the data for the field ϕ^{-1}.
Problem: Can only be done when we know the deformation fields.

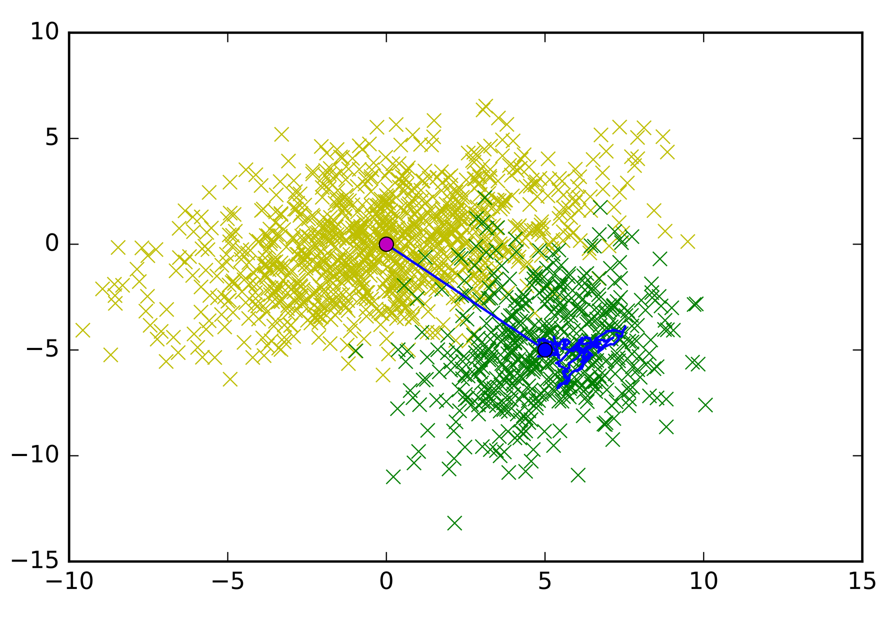

Figure 5: Matching moments of distributions. (green) the transition distribution of the blue process. (yellow) the data distribution.

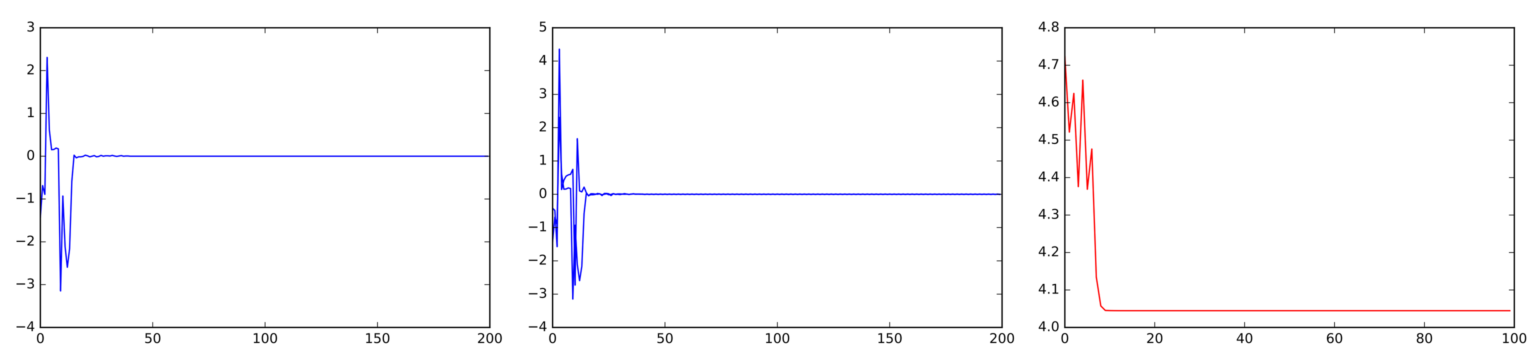

Figure 6: (left) 1. gradient, (middle) 2. gradient and (right) function values over iterations.

Preliminary Results

As a first trial we simulate a dataset of tranformed circles based on a noise field shown in Fig. 7. We will estimate the location of the noise field, σ. The noise field is modelled as a Gaussian gradient field with location $(0.6, 0.5)$ and variance 0.015. In Fig. 7 is shown two circles from the dataset simulated by the model. The estimated location for the noise field based on the method of moments is: Est.: $(0.583, 0.501)$.

Figure 7: (left) noise field, (middle) and (right) examples of simulated circles.