Multiphase Local Mean Geodesic Active Regions

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Summary
We present two similar variational methods for multiphase segmentation of complex weakly structured 3D images affected by local and global intensity inhomogeneities as is observed in micro-tomography. The methods use a fixed number of classes and utilize local image averages as region descriptors to produce per voxel posterior probabilities a la Hidden Markov Measure Field Models (HMMFM). The methods use a weighted Total Variation (wTV) and weighted Dirichlet (squared gradient) as regularizers respectively.

Our problem
We aim to segment X-ray computerised micro and nanotomograph samples of geological origin. Samples contain homogeneous materials with flat surfaces and edges, but their shapes are rather complex and not well-structured. We model images with these properties as a function \( u : \mathbb{R}^d \to \mathbb{R} \), with \( d = 2, 3 \)

\[
    u = L \left( \sum_{i=1}^{n} a_i \chi_i \right) + \eta \quad (1)
\]

where \( \Omega_i \) is the \( i \)th individual segment and \( a_i \) its intensity. \( L \) models blur and partial volume effects. \( \eta \) represents additive bias field and noise, which is considered Gaussian in high-photon-count synchrotron imaging. Figure 1 shows an example of an experimental dataset with local bias fields.

![Experimental dataset with highlighted intensity inhomogeneity areas with highlighted intensity inhomogeneity areas.](image)

Our approach
- As in our previous work, we use a local mean estimate in the data fidelity term to account for observed intensity inhomogeneities.
- The smoothing kernels can be any rotationally symmetric kernel, but we have used Gaussian and moving average kernels, of supervised extent (standard deviation and radius), based on the variation degree of the unknown bias field.
- To accomodate uncertainty in label estimates, we use a soft labelling approach, formalized by HMMFM. Additionally, to deal with partial volume effects at segment interfaces, we propose to use a Tikhonov regularization on the label field. Otherwise we use TV regularization.
- To avoid over-smoothing strong edges, we propose to locally weight the regularization by any chosen constant function, depending on the image.

Model overview
Our proposed energy formulations are

\[
    E_{\text{TV}}(c, x) = \sum_{x} \left[ g \left( x - c(x) \right)^2 \right] + \eta \quad (2)
\]

and

\[
    E_{\text{wTV}}(c, x) = \sum_{x} \left[ g \left( x - c(x) \right)^2 \right] + \frac{1}{2} [Dw(x)]^2
\]

where \( \Sigma_x \) being the standard simplex, \( ||Dw||^2 \) = \( \sum_{i=1}^{n} \int_{|x| \in \Omega_i} |\nabla w(x)|^2 \), and \( J_{\text{TV}} \) given as

\[
    J_{\text{TV}}(x) = \int_{\Omega} g(x, \nabla w(x)) \, \delta \in BV(\Omega, \Sigma_x)
\]

where \( \delta^* \) is the convex relaxation w.r.t. \( p \in \mathbb{R}^{n \times d} \) in

\[
    \psi(x, p) = \| \nabla \delta(x) \| + \delta \neq 0 \quad \| p \|_1 = 0 \quad \| p \|_1 = 1, \text{ otherwise}
\]

and \( BV(\Omega, \Sigma_x) = \{ \cdot : \Omega \to \Sigma_x, \delta \in BV(\Omega, \Sigma_x) \} \), \( BV \) being the space of weakly total variation of \( \delta \), \( J_{\text{TV}}(x) = \int_{\Omega} |\nabla w(x)|^2 \), the latter given as

\[
    J(x) = \sup \left\{ \int_{\Omega} \phi \, d\delta(x), \phi \in C(\Omega, \mathbb{R}^d), |\phi(x)| \leq L(x) \right\}
\]

Algorithm and optimization
We use an alternate approach for optimizing the two energy formulations for \( v \) and \( c \), which is outlined in Algorithm 1.

![Algorithm 1: Sketch of the algorithms.](image)

Experimental validation
We evaluate results on a synthetic volume consisting of randomly distributed balls in a volume of size 250³, with noise and bias fields, expecting 4 segments.

![Figure 2: Row 1: Synthetic data and segmentation. Row 2: Experimental data, \( E_{\text{wTV}} \), and \( E_{\text{TV}} \) segmentations respectively.](image)

Quality measures
We report Dice score index, true positive rate (TPR), true negative rate (TNR), and positive predictive value (PPV). We compare with our previous unweighted work using nearest neighbour kernels, as they slightly outperformed Gaussian kernels. We also compare with the regular piecewise constant Mumford-Shah, Otsu’s thresholding method and a dual filtering approach, that utilizes an unsharp mask and median filter before thresholding, that is popular in geosciences.

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